Closing *Tues*: 3.10
Closing *Thurs*: 4.1(1) and 4.1(2)
Exam 1 is next **Tuesday**!
covers 3.1-3.6, 3.9-3.10, 10.2, 4.1

3.10 Linear Approx. *(continued)*

Recall:

Given a point (x_0, y_0) and a curve.
The tangent line at the point can be thought of as a linear approximation:

$$y = m(x - x_0) + y_0$$

where $m = \frac{dy}{dx}$ at the point.

Entry Task:
Using tangent line approximation to estimate the value of $\sqrt[3]{8.5}$.

Note the function is $f(x) = \sqrt[3]{x}$.
Use the "nice" nearby value of x.

Example (from HW):
A cone with height *h* and base radius *r* has total surface area:

$$S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$$

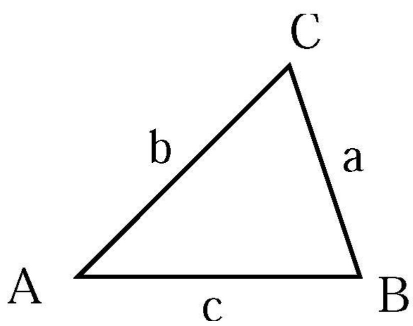
You start with *h = 8* and *r = 6*, and you want to change the dimensions in such a way that the total *surface area remains constant*.

Suppose the height increases by 26/100.

In this problem, use tangent line approximation to estimate the new value of *r* so that the new cone has the *same total surface area*.

Example (from HW):

Suppose that *a* and *b* are pieces of metal which are hinged at *C*.



You *always* have (`law of sines")

$$\frac{b}{a} = \frac{\sin(B)}{\sin(A)}$$

Initially:

angle *A* is $\pi/4$ radians= 45^o and

angle *B* is $\pi/3$ radians = 60^o.

You then widen *A* to 46^o,

without changing the sides a and b.

Use the linear approximation to estimate the new angle B.

4.1: Critical Points and Absolute Max/Min

Given $y = f(x)$.
The first questions we always ask:

1. What is the domain?
   (What inputs are allowed?)

2. What are the "critical numbers"?
   A **critical number** is a number *x = a* that is in the domain and either
   (a) $f'(a) = 0$, or
   (b) $f'(a)$ does not exist.

Example (from homework):

$$y = x^3 + 3x^2 - 72x$$

a) What is the domain?

b) What are the critical numbers?

Example:

$$f(x) = 4x + \frac{1}{x}$$

a) What is the domain?

b) What are the critical numbers?

Example:

$$g(x) = 3x - x^{1/3}$$

a) What is the domain?

b) What are the critical numbers?

Absolute Max/Min

The **absolute max** (or **global** max) is the highest *y*-value on the interval. The **absolute min** (or **global** min) is the lowest *y*-value on the interval.

Big, key, awesome observation:
(*Extreme Value Theorem*)
The absolute max/min always occur at critical numbers or endpoints!

Procedure to find absolute max/min:

1. Find critical numbers.
2. Plug endpoints and critical numbers into the function.

Example (like HW):

Find the abs. max and min of

$f(x) = x^3 + 3x^2$ on $[-1,2]$.

Small Note:
The **value** of a function, $y = f(x)$, is the output y-value. A question asking for the absolute max of a function is asking for the **y-value**. (The x-value is the location where the max *occurs*)

Example:
Find the abs. max and min of $f(x) = x\ln(x)$ on $[1, e]$.

Example:

Find the abs. max and min of

$f(x) = x\sqrt{1-x}$ on $[-1,1]$.